विद्या सर्वार्थ साधिका

# ANANDALAYA
## PRACTICE TEST
## Class : XII

Subject: Mathematics — M.M: 80

Date : 16/12/2019 — Time: 3 Hours

**General Instructions:**

(i) All the questions are compulsory.

(ii) The question paper consists of 36 questions divided into 4 sections A, B, C, and D.

(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.

(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

### SECTION-A

**Q. 1 to Q.10 are multiple choce type questions. Select the correct option.**

1. If a matrix A is both symmetric and skew symmetric then matrix A is ______. (1)

(a) a scalar matrix (b) a diagonal matrix
(c) a zero matrix of order n x n (d) a rectangular matrix

2. If matrices A and B are inverse of each other then which relation is true from the following? (1)

(a) AB = BA (b) AB = BA = I (c) AB = BA = O (d) AB = O, BA = I

3. If $(2\hat{\imath} + 6\hat{\jmath} + 14\hat{k}) \times (\hat{\imath} - \alpha\hat{\jmath} + 7\hat{k}) = \vec{0}$. then what is the value of α? (1)

(a) 3 (b) – 3 (c) 12 (d) – 12

4. Given P (A) = 0.4, P (B) = 0.7 and P (B/A) = 0.6. Find P (A ∪ B). (1)

(a) 0.86 (b) 0.76 (c) 0.14 (d) 0.24

5. A dealer wishes to purchase a number of fans and sewing machines. He has only ` 5,760 to invest and has space for at most 20 items. A fan costs him ` 360 and a sewing machine ` 240. He expects to sell a fan at a profit of ` 22 and a sewing machine at a profit of ` 18. Assuming that he can sell all the items that he buys, how should he invest his money to maximize the profit? The LPP for above equation is _____ (1)

(a) x → fans, y → sewing machines
To maximize Z = 22x + 18y
Subject to constraints
x ≥ 0, y ≥ 0, x + y ≤ 20
360 x + 240 y ≥ 5760

(b) x → fans, y → sewing machines
To maximize Z = 18x + 22y
Subject to constraints
x ≥ 0, y ≥ 0, x + y ≤ 20
360 x + 240 y ≥ 5760

(c) x → fans, y → sewing machines
To maximize Z = 18x + 22y
Subject to constraints
x ≥ 0, y ≥ 0, x + y ≥ 0
360 x + 240 y ≤ 5760

(d) x → fans, y → sewing machines
To maximize Z = 22x + 18y
Subject to constraints
x ≥ 0, y ≥ 0, x + y ≤ 20
360 x + 240 y ≤ 5760

6. If $\sec^{-1} x + \sec^{-1} y = \pi$, then the value of $\text{cosec}^{-1} x + \text{cosec}^{-1} y$ is ________. (1)

(a) 0 (b) π (c) π/2 (d) – π/2

7. Two dice are thrown. Find the probability that the numbers appeared have a sum 8 if it is known that the second die always exhibits 4. (1)
(a) 1/2 (b) 1/4 (c) 1/6 (d) 1/8

8. Value of $\int_0^1 \frac{1}{\sqrt{2x+3}} dx$. (1)
(a) $(\sqrt{3} - \sqrt{5})$ (b) $2(\sqrt{5} - \sqrt{3})$ (c) $(\sqrt{5} - \sqrt{3})$ (d) $2(\sqrt{3} - \sqrt{5})$

9. What is the distance (in units) between the two planes $2x - y + 2z = 5$ and $5x - 2.5y + 5z = 20$. (1)
(a) 1/3 (b) 1 (c) 3 (d) 13/3

10. What is the angle between the following pair of lines: (1)
$$\frac{-x+2}{-2} = \frac{y-1}{7} = \frac{z+3}{-3} \text{ and } \frac{x+2}{-1} = \frac{2y-8}{4} = \frac{z-5}{4}$$
(a) 0 (b) π/6 (c) π/2 (d) π/4

**Q. 11 to Q. 15 fill in the blanks**

11. If $f: R \rightarrow R$ is defined by $f(x) = 3x + 2$, then the value of $(fof)(-1)$ is ______. (1)

12. If the function $f(x) = \begin{cases} kx^2 & if\ x \geq 1 \\ 4 & if\ x < 1 \end{cases}$ is continues at x = 1, then the value of k is_____. (1)

13. If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric, then the values of 'a' is ____and 'b' is ____. (1)

14. The equation of the normal to the curve y = sinx at (0, 0 ) is__________. (1)

**OR**

If the radius of a circle is increased from 5 cm to 5.1 cm. The approximate increase in area is____.

15. The parallelogram having diagonals$(3\hat{\imath} + \hat{\jmath} - 2\hat{k})$ and $(\hat{\imath} - 3\hat{\jmath} + 4\hat{k})$, then the area of it is ________ sq units. (1)

**OR**

If $|\vec{a}| = 2, |\vec{b}| = \sqrt{3}$ and $\vec{a} \cdot \vec{b} = \sqrt{3}$, then the angle between $\vec{a}$ and $\vec{b}$ is ________.

**Q. 16 to Q. 20 Answer the following questions.**

16. If a, b, c are in AP, prove that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$. (1)

17. Evaluate: $\int_0^{\frac{\pi}{2}} e^x (\sin x - \cos x) dx$. (1)

18. Evaluate: $\int \frac{\cos x - \cos 2x}{1 - \cos x} dx$. (1)

**OR**

Evaluate: $\int \frac{sin^3 x + cos^3 x}{sin^2 x \cdot cos^2 x} dx$.

19. Evaluate: $\int \left( \frac{1}{\log x} - \frac{1}{(\log x)^2} \right) dx$. (1)

20. Find the differential equation of the family of lines passing though the origin. (1)

## SECTION-B

21. Write in the simplest form: $\cot^{-1}\left(\sqrt{1+x^2} - x\right)$. (2)

**OR**

Let R be the relation defined on N (set of natural numbers) defined by R = {(a, b): a, b $\in$ N and b is divisible by a}. Show that the relation is transitive. Write the equivalence class [4].

22. If $y = cosec\, x + \cot x$, show that $\sin x \times \frac{d^2y}{dx^2} = y^2$. (2)

23. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which y – coordinate is changing 8 times as fast as the x-coordinate. (2)

24. For three non-zero vectors $\vec{a}, \vec{b}\ and\ \vec{c}$, prove that $\left[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}\right] = 2\left[\vec{a}, \vec{b}, \vec{c}\right]$. (2)

**OR**

If $\vec{p} = 5\hat{\imath} + \lambda\hat{\jmath} - 3\hat{k}$ and $\vec{q} = \hat{\imath} + 3\hat{\jmath} - 5\hat{k}$, then find the value(s) of $\lambda$ so that $\vec{p} + \vec{q}$ and $\vec{p} - \vec{q}$ are perpendicular vectors.

25. Find the angle θ between the line $\frac{x-2}{3} = \frac{y-3}{5} = \frac{z-4}{4}$ and the plane $2x - 2y + z - 5 = 0$. (2)

26. A die is thrown twice and the sum of the numbers appearing is observed to be 7. What is the conditional probability that the number 2 has appeared at least once? (2)

## SECTION – C

27. Let A = R – {3} and B = R – {1}. Show that the function $f: A \rightarrow B$ defined by $f(x) = \frac{x-2}{x-3}$ is bijective. Also find its inverse. (4)

28. If $y = (\sin^{-1} x)^2, prove\ that\ (1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$. (4)

**OR**

Find the derivate of $\sec^{-1}\left(\frac{1}{2x^2 - 1}\right)\ with\ restpect\ to\ \sqrt{1 - x^2}$ at $x = ½$.

29. Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$ given that y = 0, when x =1. (4)

30. Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\cot x}}$. (4)

31. From a lot of 10 bulbs, which includes 3 defective bulbs, a sample 0f 2 bulbs is drawn at random. Find the probability distribution of defective bulbs. (4)

**OR**

Three machines E1, E2 and E3 in a certain factory producing electric bulbs, produce 50%, 25% and 25% respectively, of the total output of electric bulbs. It is know that 4% of the bulbs produced by each machines E1 and E2 are defective and that 5% of those produced by machine E3 are defective. If one bulb is picked up at random from a day's production, calculate the probability that it is defective.

32. A factory owner purchases two types of machines A and B for his factory. The requirements and limitations for the machines are as follows: (4)

| *Machine* | *Area occupied by the machine* | *Labour force for each machine* | *Daily output (in unit)* |
|---|---|---|---|
| *A* | *1000 sq. m* | *12 men* | *60* |
| *B* | *12 sq. m* | *8* | *40* |

He has an area of 9000 sq. m available and 72 skilled men who can operate the machines. How many machines of each type should he buy to maximize the daily output?

**SECTION-D**

33. Using the properties of determinants, prove that (6)

$$\begin{vmatrix} a^2+1 & ab & ac \\ ba & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = a^2+b^2+c^2+1.$$

**OR**

If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find AB. Hence, solve the system of equations:

$x - y = 3, 2x + 3y + 4z + 17 \; and \; y + 2z = 7.$

34. Using integration, find the area of the region $\left\{(x, y);\ x^2 + \ y^2 \leq 1\ \leq x + \frac{y}{2}\right\}$. (6)

35 Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible, when revolved about one of its sides. Also find the maximum volume. (6)

**OR**

AB is a diameter of a circle and C is any point on the circle. Show that the area of triangle ABC is maximum, when it is isosceles.

36. Find the coordinates of the point where the line through the points A (3, 4, 1) and B (5, 1, 6) crosses the plane determined by the points P (2, 1, 2), Q (3, 1, 0) and R (4, – 2, 1). (6)